PD Control of Closed-Chain Mechanical Systems: An Experimental Study

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Abstract

In this paper, we briefly review recent results by the authors on the formulation of the equations of motion of closed-chain mechanisms and the development of PD control strategies with guaranteed Lyapunov asymptotic stability. We then introduce and derive the equations of motion of the Rice Planar Delta Robot which was designed and built at Rice University as a test bed to perform control experiments for closed-chain mechanisms. Finally, we present simulation as well as experimental results to illustrate the successful application of PD plus simple gravity control strategy for this class of dynamical systems.

Keywords: Closed-Chain Mechanisms, Parallel Robots, Dynamics, PD Control, Experimental Study

1 Introduction

In most robot manipulators that are popular today, the links are connected sequentially by actuated joints. For these robot manipulators, which we refer to as serial robots (or open-chain mechanisms), a well established formulation of the equations of motion exist [11] and a wealth of control results have been developed during the last two decades (see [8] and [1] for recent surveys). Closed-chain mechanisms, sometimes referred to as parallel robots, are characterized by having the links connected in series and parallel combinations and only some of the joints are actuated (refer to Figure 1). In many cases, the actuators are placed lower (closer to the base or on the base itself) in the link chain. This makes the moving parts lighter which can lead to greater efficiency and faster acceleration at the end-effector. Closed-chain mechanisms offer also greater rigidity to weight ratio. Therefore, they are more suitable for fast assembly lines. An early example of a parallel robot is the Stewart platform [12]. More recently developed parallel robots include the Delta robot [3] and its extension the Hexa robot [10]. Unlike serial robots, the derivation of dynamic equations of motion for closed-chain mechanisms suitable for the design of appropriate control strategies is still not well developed due to the complexity of the kinematics and dynamics analyses.

In [6], a reduced model for parallel robots in terms of independent generalized coordinates was developed. The main advantage of this formulation is that the final equations of motion are in the so-called reduced form which enables the possibility of the extension of the existing wealth of serial robot control results to closed-chain mechanisms. In fact, it was shown in [5] that due to a skew symmetry property similar to the case of serial robots, PD control strategies can insure Lyapunov asymptotic stability.

In this paper, we first review in Section 2 the formulation of the equations of motion of closed-chain mechanisms and the stability properties of a PD control law originally developed in [6] and [5]. We discuss the computational and implementation characteristics of the dynamics model and the control law. In Section 3, we introduce the Rice Planar Delta Robot (R.P.D.R.) which is a robot designed and built at Rice University as a test bed to perform control experiments on closed-chain mechanisms. We develop the equations of motion of the R.P.D.R. which we use in a later section to set up a simulation experiment. In Section 4, we implement a PD plus simple gravity control law of [5] on the R.P.D.R. and compare the experimental results with the simulation results which both confirmed the theoretical predictions in [5]. Finally in Section 5, we present conclusions.
2 PD Control of Closed-Chain Mechanisms

In this section we first review the dynamics model for closed-chain mechanisms originally presented in [6], and second summarize the PD plus simple gravity control result for this class of systems originally developed in [5]. This will be used as the basis to develop the equations of motion for the R.P.D.R. and implement the control result experimentally in later sections.

2.1 A Reduced Model for Closed-Chain Mechanisms

A closed-chain mechanical system, referred to here as a constrained system $\Sigma$, can be thought of as consisting of a free system $\Sigma'$ to which constraints $C$ are applied as shown in Figure 2 for an illustrative planar mechanism. The free system $\Sigma'$ is an $n'$ degree-of-freedom (d.o.f) holonomic system which consists of a collection of rigid bodies described by the following differential equation

$$
\Sigma' : \quad D'(q') \dot{q}' + C'(q', \dot{q}') \dot{q}' + g'(q') = 0
$$

where $q' \in \Omega'$ and $\Omega' \subset \mathbb{R}^{n'}$. $q'$ is the vector of the generalized coordinates of the free system $\Sigma'$, $D'(q') \in \mathbb{R}^{n' \times n'}$ is the inertia matrix, $C'(q', \dot{q}') \dot{q}' \in \mathbb{R}^{n'}$ represent the centrifugal and Coriolis terms, and $g'(q') \in \mathbb{R}^{n'}$ is the gravity vector. The constraints applied at the free system are represented by $(n' - n)$ independent scleronomic holonomic constraints given by

$$
C : \quad \phi(q') = 0
$$

where $\phi(q')$ is at least twice continuously differentiable, a consequence of which is that $\phi_{\dot{q}'}(q') = \frac{\partial}{\partial \dot{q}'} \phi(q')$ is of full $(n' - n)$ rank. With the introduction of the constraints (2), the generalized coordinates $\hat{q}'$ are restricted to a subspace of $\Omega'$, namely, $\hat{q}' \in U'$ where

$$
U' \doteq \{ q' \in \Omega' : \phi(q') = 0 \} \subset \Omega'.
$$

It follows from standard results in dynamics [7] that the free system $\Sigma'$ (1), with imposed constraints $C$ (2), has $n$-d.o.f, and hence there exists a minimum set of $n$-independent generalized coordinates $q \in \Omega \subset \mathbb{R}^{n}$ such that the system can be written in terms of $q$ as follows

$$
\Sigma : \quad D(q) \dot{q} + C(q, \dot{q}) \dot{q} + g(q) = 0
$$

where $D(q) \in \mathbb{R}^{n \times n}$ is the inertia matrix, $C(q, \dot{q}) \dot{q} \in \mathbb{R}^{n}$ is the centrifugal and Coriolis vector, and $g(q) \in \mathbb{R}^{n}$ is the gravity vector. This formulation is sometimes referred to as formulation in reduced form [15]. In many situations, such as the case of the R.P.D.R., we may be able to choose the generalized coordinates to coincide with the variables of the actuated joints in which case the equations of motion (4) can be written as

$$
\Sigma : \quad D(q) \dot{\hat{q}} + C(q, \dot{q}) \dot{\hat{q}} + g(q) = u
$$

where $u \in \mathbb{R}^{n}$ is the applied generalized force vector.

Using the given information about the free system $\Sigma'$ and the constraints $C$, a reduced model $\Sigma$ was developed in [6] and summarized in the following steps:

- The independent generalized coordinates $q$ with which we would like to describe the constrained system (4) can be chosen to satisfy the following twice continuously differentiable parameterization

$$
C(q) \doteq \phi(q)
$$

Define the following quantities $\psi(q') \doteq \phi(q')$:

$$
\psi(q') \doteq \frac{\partial}{\partial \dot{q}'} \phi(q') \doteq \left[ \frac{\phi(q')}{\dot{q}'} \right] - \frac{\partial}{\partial \dot{q}'} \phi(q') = v(q')
$$

$\psi(q')$ can be defined by the set $v(q') \doteq \psi(q')$. Note that $v(q') \in \mathbb{R}^{n'}$. We now define the set $V' \doteq \{ q' \in U' : \psi(q') \neq 0 \} \subset U'$. We say that the system is in a singular configuration when $q'$ is an element of $U$' but is not an element of $V'$. It follows that $V'$ is the workspace region in the $q'$ coordinates where the constrained system satisfies the constraints and in addition is not in a singular configuration.

- For any given point $q' \in V'$, let $\psi(q', q_0) = 0$. Using the Implicit Function Theorem [14], we conclude that there exist a neighborhood $N_{q'}$ of $q'$, and a neighborhood $N_{q_0}$ of $q_0$, such that for any $q \in N_{q_0}$, there exists a unique $q' \in N_{q'}$ such that $q' = \sigma(q)$. Furthermore, $q' = \rho(q') \dot{q}$ where

$$
\rho(q') = \psi^{-1}(q') \left[ \begin{array}{c} 0_{n' - n} \\ I_{n \times n} \end{array} \right].
$$

We now let $W' \subset V'$ denote the largest subset of $V'$ containing $q'$ for which the unique parameterization $q' = \sigma(q)$ holds. An explicit characterization of $W'$ for the general case is reported in [2]. We now denote the corresponding domain of $\sigma$ by $\Omega$. Hence, we have a diffeomorphism from $W'$ to $\Omega$ as follows:

$$
W' \rightarrow \theta \rightarrow \Omega \rightarrow \sigma, W' \rightarrow W'.
$$

Finally, the equations of motion of the constrained system expressed in terms of independent generalized coordinates $q \in \Omega$, as given in (5) and repeated here for convenience,

$$
\Sigma : \quad D(q) \dot{q} + C(q, \dot{q}) \dot{q} + g(q) = u
$$

are obtained by combining

$$
\begin{align*}
\{ & D(q') \dot{\hat{q}} + C(q', \dot{\hat{q}}) \dot{\hat{q}} + g(q') = u \\
\{ & q' = \sigma(q) \\
\{ & q' = \rho(q') \dot{q}
\end{align*}
$$

Figure 2: Free System, Constraints, and Constrained System
where \( \forall \mathbf{q} \in \mathbb{W} \)

\[
D(\mathbf{q}') = \mathbf{\rho}(\mathbf{q}')^T D(\mathbf{q}) \mathbf{\rho}(\mathbf{q}) \\
C(\mathbf{q}', \mathbf{\dot{q}}') = \mathbf{\rho}(\mathbf{q}')^T C'_{\mathbf{q}' \mathbf{\dot{q}}'} \mathbf{\rho}(\mathbf{q}) + \mathbf{\rho}(\mathbf{q}')^T D'(\mathbf{q}) \mathbf{\rho}(\mathbf{q}, \mathbf{\dot{q}}) \\
\mathbf{g}(\mathbf{q}) = \mathbf{\rho}(\mathbf{q}')^T \mathbf{g}(\mathbf{q}') \\
\mathbf{\rho}(\mathbf{q}) = \text{is given by equation (6)}
\]

(9) 
(10) 
(11) 
(12)

The reduced model described above has two special characteristics which make it different from regular models of open-chain mechanical systems. First, the above reduced model is valid only (locally) in a compact set \( \Omega \). This means that control strategies can only insure local stability results at best. Second, since the parameterization \( \mathbf{q}' = \sigma(\mathbf{q}) \) is implicit, it is an implicit model. It follows that in order to apply model based control strategies, there is a need to instantaneously compute the parameterization \( \mathbf{q}' = \sigma(\mathbf{q}) \) which is a difficult task in general even though its existence is insured by the Implicit Function Theorem. When implementing model based control laws using digital computers, this means that at each fraction of the control sampling period, a numerical technique such as Newton-Raphson has to successfully compute this parameterization which, even with the current advances in digital hardware, could be very difficult to achieve. This difficulty is avoided with a PD plus simple gravity compensation control as discussed next.

### 2.2 PD Plus Simple Gravity Compensation

The applicability of PD-based control strategies for closed-chain mechanisms was investigated in [5]. It was proposed that PD with full as well as with simple gravity compensation were possible thanks to the skew symmetry property of \( D - 2C \), a property that was established for closed-chain mechanisms in [5]. In particular, PD plus simple gravity compensation is very attractive because it avoids the online difficulties is avoided with a PD plus simple gravity compensation law.

\[ \mathbf{u} = K_P (\mathbf{q}^d - \mathbf{q}) - K_V \mathbf{\dot{q}} + \mathbf{g}(\mathbf{q}^d). \]

(13)

In reference to the equations of motion of the closed-chain mechanism, equation (8), note that

\[ \mathbf{g}(\mathbf{q}^d) = \mathbf{\rho}^T (\sigma(\mathbf{q}^d)) \mathbf{g}' (\sigma(\mathbf{q}^d)). \]

Since the parameterization \( \mathbf{q}' = \sigma(\mathbf{q}^d) \) is now a constant value, it could be computed off-line to any degree of accuracy using any of the many well established numerical techniques. It follows that the constant term \( \mathbf{g}(\mathbf{q}^d) \) in (13) could also be accurately computed off-line.

Suppose now that for a particular closed-chain system, there is a simply connected set \( \mathcal{D} \subset \Omega \) and consider a setpoint (regulation) problem in which it is required that the vector \( \mathbf{q} \) reach a constant desired value \( \mathbf{q}_d \in \mathcal{D} \). Define \( \mathbf{q} \triangleq \mathbf{q}_d - \mathbf{q} \), and \( \mathcal{B} \triangleq \{(\mathbf{q}, \mathbf{\dot{q}}) : \mathbf{q}, \mathbf{\dot{q}} \in \mathcal{D} \} \). The PD plus simple gravity compensation control result is summarized as follows:

**Theorem 2.1** [5] Define the set

\[ \mathcal{B}_2 \triangleq \{(\mathbf{q}, \mathbf{\dot{q}}) : V_2(\mathbf{q}, \mathbf{\dot{q}}) \leq c_2 \} \] where \( c_2 \) is the largest positive real number such that \( \mathcal{B}_2 \subset \mathcal{B} \). Let the initial conditions \( (\mathbf{q}_0, \mathbf{\dot{q}}_0) \in \mathcal{B}_2 \), and let \( \|\mathbf{g}(\mathbf{q})\| \leq \beta \) in \( \mathcal{B}_2 \) where \( \beta \)

is a positive constant. Choose \( k_\mu > \beta, i = 1, 2, \ldots, n \), where \( k_\mu \) are the diagonal elements of \( K_\mu \). Then the equilibrium \( \mathbf{q} = 0 \) and \( \mathbf{\dot{q}} = 0 \) of the closed-chain mechanism model (8) with the control law (13) is (locally) asymptotically stable and \( \mathbf{q} \longrightarrow \mathbf{q}_d \), \( \mathbf{\dot{q}} \longrightarrow 0 \), as \( t \longrightarrow \infty \).

Note that the existence of the constant \( \beta \) is insured by the continuity of the gravity vector \( \mathbf{g} \) since the set \( \mathcal{B}_2 \) is compact.

### 3 The Rice Planar Delta Robot (R.P.D.R.)

The R.P.D.R. was designed and built at Rice University as a test-bed to perform experiments on parallel robots (See Figure 3). The link configuration is shown in Figure 4. It has four links connected through revolute joints. Two of the links (Link 1 and Link 2) are actuated with DC motors while the other two links (Link 3 and Link 4) are passive. It can be seen that the R.P.D.R. has two degrees of freedom. Thus, the number of inputs are selected to match the number of degrees of freedom. The R.P.D.R. is controlled with an IBM PC to which it is interfaced through a DSP board made by dSPACE [4]. Position feedback of Link 1 and Link 2 are provided with optical encoders. Velocity feedback is provided by differentiating the position and filtering with a low pass filter implemented in software. The dSPACE board computes the control law in real time based on the position/velocity feedback and outputs the desired torque at the motors.

**Figure 3:** Rice Planar Delta Robot (R.P.D.R.)

**Figure 4:** Link and Joint Configuration of the R.P.D.R.
ous section. The first step in deriving the equations of motion is selecting the “free system”. In our free system, the robot is virtually cut open at the end-effector, resulting in two serial robots each having two degrees of freedom (see Figure 5). As defined in the figure, $m_i$, $a_i$, and $a_i$ are respectively the mass, distance to the center of mass, and length of link $i$. The inertia of link $i$ about the line through the center of mass parallel to the axis of rotation is denoted by $I_i$. The parameters corresponding to Link 1 and Link 3 are defined similar to the parameters of Link 2 and Link 4 even though they are not shown in the figure. Thus the constraint equations are due to point $E$ being coincident with point $F$ and are given by,

$$\phi(q') = \begin{bmatrix} \phi(1) \\ \phi(2) \end{bmatrix} = 0, \quad (14)$$

where $\phi(1) = a_1\cos(q_1) + a_2\cos(q_1 + q_2) - a_2\cos(q_2) - a_3\cos(q_2 + q_3)$ and $\phi(2) = a_2\sin(q_1) + a_3\sin(q_1 + q_2) - a_2\sin(q_2) - a_3\sin(q_2 + q_3)$. The vector $q' = [q_1 \ q_2 \ q_3] \ T$ is the generalized coordinate vector of the free system. Note that since the actuated joints are joints 1 and 2 for the R.P.D.R., we choose the generalized coordinate vector of the constrained system to be $q = [q_1 \ q_2 \ q_3] \ T$. Our next objective will be to derive expressions for each of the terms appearing in (8) for the R.P.D.R. to obtain the equations of motion. We begin with the parameterization $\alpha(q') = q$, which is given by,

$$\alpha(q') = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad (15)$$

Now by combining (14) and (15) and differentiating with respect to $q'$, we obtain the following expression for $\psi_q(q')$:

$$\psi_q(q') = \begin{bmatrix} \psi_{q'}(1, 1) & \psi_{q'}(1, 2) & \psi_{q'}(1, 3) & \psi_{q'}(1, 4) \\ \psi_{q'}(2, 1) & \psi_{q'}(2, 2) & \psi_{q'}(2, 3) & \psi_{q'}(2, 4) \end{bmatrix}, \quad (16)$$

where, $\psi_{q'}(1, 1) = -a_2\sin(q_1) - a_3\sin(q_1 + q_2)$, $\psi_{q'}(1, 2) = a_2\sin(q_2) + a_3\sin(q_2 + q_3)$, $\psi_{q'}(1, 3) = -a_2\sin(q_2 + q_3)$, $\psi_{q'}(1, 4) = a_2\sin(q_2 + q_3)$, $\psi_{q'}(2, 1) = a_3\cos(q_1) + a_3\cos(q_1 + q_3)$, $\psi_{q'}(2, 2) = -a_3\cos(q_2) - a_3\cos(q_2 + q_3)$, $\psi_{q'}(2, 3) = a_3\cos(q_2 + q_3)$, and $\psi_{q'}(2, 4) = -a_3\cos(q_2 + q_3)$. Now from (6) we have the following expression for $\rho(q')$:

$$\rho(q') = \psi^{-1}_q(q') \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}. \quad (17)$$

Since $\rho(q')$ is in terms of an inverse matrix, it is not easy to take the time derivative. Therefore, we use the following expression for $\rho(q', q')$, which can be easily obtained by pre-multiplying (17) with $\psi_q(q')$ and taking the time derivative:

$$\rho(q', q') = -\psi^{-1}_q(q') \psi_q(q', q') \rho(q'), \quad (18)$$

where $\psi_q(q', q')$ can be obtained by differentiating (16) with respect to time. Next we will derive expressions for $D(q')$, $C^g(q', q')$, and $g^g(q')$ that appear in (9), (10), and (11). The following expressions were derived using the Lagrangian method:

$$D(q') = \begin{bmatrix} d_{1,1} & d_{1,3} & 0 \\ d_{1,1} & d_{2,3} & 0 \\ d_{1,1} & d_{2,1} & d_{4,4} \end{bmatrix}, \quad (19)$$

where $d_{1,1} = m_3l_2^2 + m_3(a_1^2 + l_2^2 + 2a_1l_2\cos(q_1)) + I_1 + I_1$, $d_{1,3} = m_3(l_2^2 + a_1l_2\cos(q_1)) + I_1$, $d_{2,2} = m_4l_2^2 + m_4(a_1^2 + l_2^2 + 2a_1l_2\cos(q_1)) + I_2 + I_4$, $d_{4,4} = m_4(l_2^2 + a_1l_2\cos(q_1)) + I_4$, $d_{1,3} = d_{3,1}$, $d_{3,3} = m_3l_2^2 + I_3$, $d_{4,2} = d_{2,4}$, $d_{4,4} = m_4l_2^2 + I_4$.$$

$$C^g(q', q') = \begin{bmatrix} h_1q_1 & 0 & h_1q_2 + q_3 \\ 0 & h_1q_1 & h_1q_2 + q_3 \\ -h_1q_1 & 0 & h_1q_2 + q_3 \end{bmatrix}, \quad (20)$$

where $h_1 = -m_3a_1l_2\sin(q_1)$, and $h_2 = -m_4a_2l_4\sin(q_4)$, and

$$g^g(q') = \begin{bmatrix} (m_1l_3 + m_2a_1l_3\cos(q_1)) + m_3a_1l_2\cos(q_1 + q_3) \\ (m_2l_2 + m_4a_2l_4\cos(q_4)) + m_4l_2\cos(q_4) + q_3 \\ m_4l_2\cos(q_4) + q_3 \\ m_4l_2\cos(q_4) \end{bmatrix} g, \quad (21)$$

where $g = 9.81 \text{ m/sec}^2$ is the gravitational acceleration constant. At this point we have derived the equations of motion of the R.P.D.R. in terms of $q$. The only remaining issue is the derivation of the parameterization $q' = \sigma(q)$. In general, it is not possible to derive an analytic expression for $\sigma(q)$, and it must be computed using numerical methods. For the R.P.D.R. however, it is possible to solve (14) to obtain the following:

$$q_3 = \tan^{-1}\left(\frac{\pm \sqrt{A(q_1, q_2)^2 + B(q_1, q_2)^2 - C(q_1, q_2)^2}}{C(q_1, q_2)}\right)$$

$$+ \tan^{-1}\left(\frac{B(q_1, q_2)}{A(q_1, q_2)}\right) - q_2. \quad (22)$$

where $A(q_1, q_2) = 2a_1\lambda(q_1, q_2)$, $B(q_1, q_2) = 2a_4\mu(q_1, q_2)$, and $C(q_1, q_2) = a_1^2 - a_1^2 - \lambda(q_1, q_2)^2 + \mu(q_1, q_2)^2$, and $\lambda(q_1, q_2) = a_2\cos(q_2) - a_3\cos(q_1 + q_3)$, and $\mu(q_1, q_2) = a_3\cos(q_2 + q_3)$. Finally, $\nu_3 = \tan^{-1}\left(\frac{\nu_1(q_1, q_2) + a_3\sin(q_2 + q_3)}{\lambda(q_1, q_2) + a_4\cos(q_2 + q_3)}\right) - \nu_1. \quad (23)$

Hence, (22) and (23) combined represent the parameterization $q' = \sigma(q)$. This completes the derivation of the equations of motion of the R.P.D.R. In summary, the equations of motion of the R.P.D.R. are given by:

$$\begin{cases} D(q')q + C^g(q', q')q + g^g(q') = u, \\ \dot{q}' = \rho(q')q, \\ q' = \sigma(q). \end{cases}$$
where

\[
D(q') = \rho(q')^T D'(q') \rho(q'),
\]

\[
C(q', q^*) = \rho(q')^T C'(q', q^*) \rho(q') + \rho(q')^T D'(q') \rho(q', q^*),
\]

\[
g(q') = \rho(q')^T g'(q').
\]

\[
D'(q'), C'(q', q^*), g'(q'), \rho(q', q^*), \rho(q'), \text{and } \sigma(q) \text{ are defined in (19), (20) (21), (18), (17), and (22) plus (23) respectively.}
\]

4 Implementation of the PD Control Law

Our next objective is to implement the PD plus simple gravity compensation control discussed in an early section using the computer controlled R.P.D.R. First, we need to characterize a compact domain, a subset of \( \Omega \), where the parameterization \( \sigma(q) \) exists. For the R.P.D.R., if we select a simply connected region \( \mathcal{D} \subset \Omega \) that is free of singularities, the parameterization \( \sigma(q) \) will exist in that region. Therefore, our first objective was to characterize the singular points of the R.P.D.R. From (16), we see that \( \det[\sigma(q')] = \sin(q_1 + q_2 - q_2 - q_1) = n\pi \), for \( n = 0, \pm 1, \pm 2, \ldots \). By substituting these into the constraint equations (14), we were able to solve for all of the singular points of the R.P.D.R. These are plotted in Figure 6 which shows the entire workspace of the R.P.D.R. (corresponding to the range \(-180^\circ \) to \(180^\circ \) for Joint 1 and \(0^\circ \) to \(360^\circ \) for Joint 2). We considered two cases corresponding to \( n \) being even and \( n \) being odd. The two cases led to two types of singularities. There are infinitely many singularities of the first type which occur when \( n \) is odd. The second type occurs when \( n \) is even and the robot loses control of Link 3 and Link 4 in these configurations. There are two singularities of this type for the R.P.D.R. as shown in Figure 6. For our experiments we chose the singularity free region \( \mathcal{D} \) where \(-30^\circ \leq q_1 \leq 30^\circ \) and \(120^\circ \leq q_2 \leq 180^\circ \).

We next present some of the initial experiments that were performed to identify the system parameters and in particular frictional effects. We then show simulation as well as experimental results.

4.1 System Parameter Identification

Before implementing control, initial experiments were performed to identify the parameters in the equations of motion derived in Section 3.1. The link parameters were calculated and verified by measurement. These are presented in Table 1. In addition to the inertia represented in \( D(q') \), there is a constant inertia due to the motors and the transmission on Joint 1 and Joint 2. These constant inertias were measured to be \(3.326 \times 10^{-3} \text{ kg m}^2 \). The distance between the axes of Joint 1 and Joint 2 was measured to be \( c = 0.3048 \text{ m} \).

Our next objective was to identify the friction in the system. We considered Coulomb friction, viscous friction and static friction at the initiation of motion. Using the model of the R.P.D.R. derived in Section 2.1 together with the parameters which we calculated, we were able to setup a simulation. Friction was also included in the model with an initial guess of the friction parameters. The real values of the friction parameters were identified by comparing the actual step response of the system with the predicted response of the simulation for different step inputs. These experiments showed us that the frictional effects cannot be neglected in the R.P.D.R.

4.2 Experimental Results and Simulations

For our experiment, we chose the point-to-point trajectory shown in Figure 7 which is within the singularity free region \( \mathcal{D} \). The initial configuration is given by \( q_1 = -26^\circ \) and \( q_2 = 122^\circ \) while the final configuration is given by \( q_1 = 30^\circ \) and \( q_2 = 180^\circ \). Our next objective was to compute the parameters of the PD with simple gravity compensation control law (13), namely,

\[
u = g(q^*) + K_P(q^* - q) - K_V \dot{q}.
\]

Based on the system parameters identified in Section 4.1, we computed the gravity vector at the desired configuration \( g(q^*) \) off-line. In order to account for the frictional effects, we estimated the Coulomb friction at the desired configuration and added that to \( g(q^*) \). Since the effect of viscous friction is to increase \( K_V \), there is no need to account for it. We selected \( K_P \) to be diagonal with each element equal to \(10 \text{ N-m/rad} \), which was sufficient to satisfy the requirements of Theorem 2.1 for this experiment. The matrix \( K_V \) was also selected diagonal with the first element \(0.65 \) and the second element \(0.6 \text{ N-m/s/rad} \). The values of the elements of \( K_V \) were selected by trial and error to make the system close to critically damped.
In order to compare the experimental results with the theoretical predictions, we performed a simulation. During the simulation we integrated the equations of motion derived in Section 3.1 combined with the control law (13), to generate the solution trajectories. We used the Runge-Kutta algorithm in Matlab [9] to perform the integration. For the simulation, we had the same initial configuration and final desired configuration that was used for the experiments. We also included friction (identified in Section 4.1) in our model. The simulation and experimental results are presented in Figure 8. The dashed curves correspond to the simulation results while the solid curves correspond to the experimental results.

The simulation and experimental results were presented and showed satisfactory agreement.

5 Conclusions

In this paper, we first reviewed a new formulation of the equations of motion of closed chain mechanisms of [6]. We also reviewed a local asymptotic stability of the PD plus simple gravity compensation control law that was established in [5]. The main contribution of this paper is the experimental validation of the above results using the Rice Planar Delta Robot (R.P.D.R.) which was designed and built at Rice University as a test-bed to perform control experiments on closed-chain mechanisms. A detailed derivation of the equations of motion was also presented in this paper which enabled setting up a simulation experiment. Both simulation and experimental results were presented and showed satisfactory agreement.

References


